

The absolute mass of neutrino and the first unique forbidden β -decay of ^{187}Re

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The planned rhenium β -decay experiment, called the “Microcalorimeter Arrays for a Rhenium Experiment” (MARE), might probe the absolute mass scale of neutrinos with the same sensitivity as the Karlsruhe tritium neutrino mass (KATRIN) experiment, which will take commissioning data in 2011 and will proceed for 5 years. We present the energy distribution of emitted electrons for the first unique forbidden β -decay of ^{187}Re . It is found that the p -wave emission of electron dominates over the s -wave. By assuming mixing of three neutrinos the Kurie function for the rhenium β -decay is derived. It is shown that the Kurie plot near the endpoint is within a good accuracy linear in the limit of massless neutrinos like the Kurie plot of the superallowed β -decay of ^3H .

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I. INTRODUCTION

The recent atmospheric, solar, reactor and accelerator neutrino experiments have convinced us that neutrinos are massive particles. However, the problem of absolute values of their masses is still waiting for a solution. Neutrino oscillations depend on the differences of neutrino masses, not on their absolute values. Apparently three kinds of neutrino experiments have a chance to determine the light neutrino masses: i) cosmological measurements, ii) the tritium and rhenium single β -decay experiments, iii) neutrinoless double β -decay experiments.

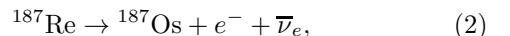
The measurement of the electron spectrum in β -decays provides a robust direct determination of the values of neutrino masses. In practice, the most sensitive experiments use tritium β -decay, because it is a super-allowed transition with a low Q -value. The effect of neutrino masses m_k ($k=1,2,3$) can be observed near the end point of the electron spectrum, where $Q - T \sim m_k$. T is the electron kinetic energy. A low Q -value is important, because the relative number of events occurring in an interval of energy ΔT near the endpoint is proportional to $(\Delta T/Q)^3$.

The current best upper bound on the effective neutrino mass m_β given by,

$$m_\beta = \sqrt{\sum_{k=1}^3 |U_{ek}|^2 m_k^2}, \quad (1)$$

has been obtained in the Mainz and Troitsk experiments: $m_\beta < 2.2$ eV [1]. U_{ek} is the element of neutrino mixing matrix. In the near future, the Karlsruhe tritium neutrino mass (KATRIN) experiment will reach a sensitivity of about 0.2 eV [2]. In this experiment the β -decay of tritium will be investigated with a spectrometer taking advantage of magnetic adiabatic collimation combined with an electrostatic filter.

Calorimetric measurements of the β -decay of rhenium where all electron energy released in the decay is recorded, appear complementary to those carried out with spectrometers. The unique first forbidden β -decay,



is particularly promising due to its low transition energy of ~ 2.47 keV and the large isotopic abundance of ^{187}Re (62.8%), which allows the use of absorbers made with natural rhenium. Measurements of the spectra of ^{187}Re have been reported by the Genova and the Milano/Como groups (MIBETA and MANU experiments). The achieved sensitivity of $m_\beta < 15$ eV was limited by statistics [1]. The success of rhenium experiments has encouraged the micro-calorimeter community to proceed with a competitive precision search for a neutrino mass. The ambitious project, called the “Microcalorimeter Arrays for a Rhenium Experiment” (MARE), is planned in two steps, MARE I and MARE II. MARE I might reach a statistical sensitivity of 4 eV after 3 years of data taking [3]. MARE II is to challenge the KATRIN goal of 0.2 eV [3].

The aim of this paper is to derive the form of the endpoint spectrum of emitted electrons for the β -decay of ^{187}Re , which is needed to extract the effective neutrino mass m_β or to place a limit on this quantity from future MARE I and II experiments.

II. FIRST UNIQUE FORBIDDEN β -DECAY OF ^{187}Re

The ground-state spin-parity is $5/2^+$ for ^{187}Re and $1/2^-$ for the daughter nucleus ^{187}Os , and the decay is associated with $\Delta J^\pi = 2^-$ ($\Delta L = 1$, $\Delta S = 1$) of the nucleus, *i.e.*, classified as first unique forbidden β -decay. The emitted electron and neutrino are expected to be,

respectively, in $p_{3/2}$ and $s_{1/2}$ states (see Appendix A) or vice versa. The emission of higher partial waves is strongly suppressed due to a small energy release in this nuclear transition.

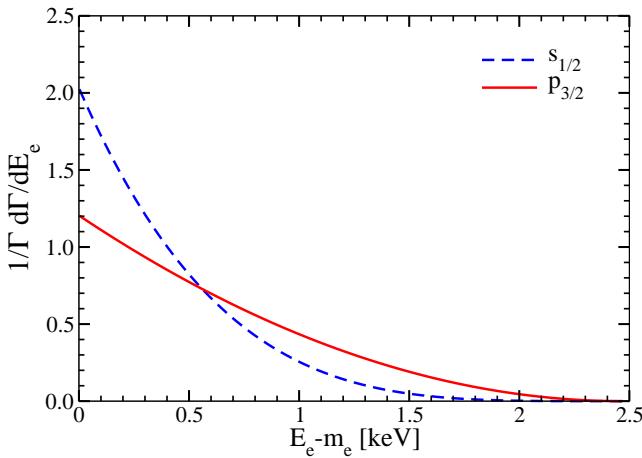


FIG. 1. (Color online) The single electron differential decay rate normalized to the particular decay rate ($\Gamma^{s_{1/2}}$ or $\Gamma^{p_{3/2}}$) for emission of $s_{1/2}$ and $p_{3/2}$ electrons vs. electron energy E_e for β -decay of ^{187}Re .

The differential decay rate is a sum of two contributions associated with emission of the $s_{1/2}$ and the $p_{3/2}$ state electrons. By considering the finite nuclear size effect the theoretical spectral shape of the β -decay of ^{187}Re is

$$\begin{aligned} \frac{d\Gamma}{dE_e} &= \frac{d\Gamma^{p_{3/2}}}{dE_e} + \frac{d\Gamma^{s_{1/2}}}{dE_e} \\ &= \sum_{k=1}^3 |U_{ek}|^2 \frac{G_F^2 V_{ud}^2}{2\pi^3} B R^2 p_e E_e (E_0 - E_e) \\ &\quad \times \frac{1}{3} [F_1(Z, E_e) p_e^2 + F_0(Z, E_e)((E_0 - E_e)^2 - m_k^2)] \\ &\quad \times \sqrt{(E_0 - E_e)^2 - m_k^2} \theta(E_0 - E_e - m_k) \quad (3) \end{aligned}$$

with

$$B = \frac{g_A^2}{6R^2} |\langle 1/2^- \| \sum_n \tau_n^+ \{\boldsymbol{\sigma}_n \otimes \mathbf{r}_n\}_2 \| 5/2^+ \rangle|^2. \quad (4)$$

G_F is the Fermi constant and V_{ud} is the element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. p_e , E_e and E_0 are the momentum, energy, and maximal endpoint energy (in the case of zero neutrino mass) of the electron, respectively. $F_0(Z, E)$ and $F_1(Z, E)$ in Eq. (3) are relativistic Fermi functions and $\theta(x)$ is a theta (step) function. g_A denotes an axial-vector coupling constant. \mathbf{r}_n is a coordinate of the n-th nucleon and R is a nuclear radius. The value of nuclear matrix element B in Eq.

(4) can be determined from the measured half-life of the β -decay of ^{187}Re .

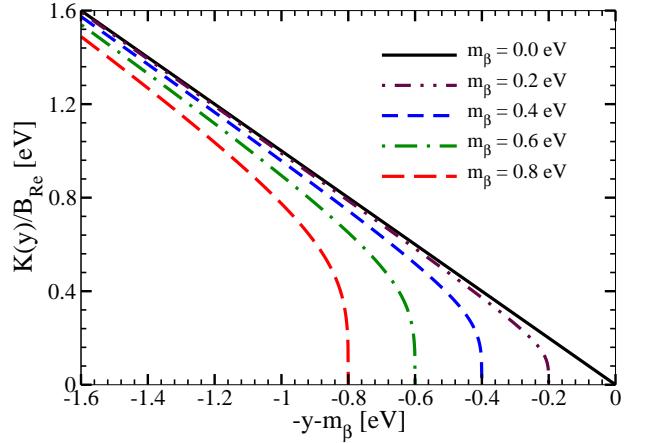


FIG. 2. (Color online) Endpoints of the Kurie plot of the rhenium β -decay for various values of the effective neutrino mass: $m_\beta = 0, 0.2, 0.4, 0.6$ and 0.8 eV.

The β -decay rate of rhenium is a sum of decay rates for emission of $p_{3/2}$ and $s_{1/2}$ electrons [see Eq. (3)]. In Fig. 1 we show the single electron differential decay rate normalized to the particular decay rate as a function of electron energy E_e . The two possibilities offer a different energy distribution of outgoing electrons. Close to the end point, there is a more flat distribution for $s_{1/2}$ electrons due to the dependence on squared neutrino momentum $p_\nu^2 = (E_0 - E_e)^2 - m_k^2$ [see Eq. (3)]. Because of this factor the two particular decay rates depend on the neutrino mass in a different way.

Experimentally, it was found that $p_{3/2}$ -state electrons are predominantly emitted in the β -decay of ^{187}Re [4]. By performing numerical analysis of partial decay rates associated with emission of the $p_{3/2}$ and the $s_{1/2}$ electrons (terms with $F_1(Z, E_e)$ and $F_0(Z, E_e)$ in Eq. (3), respectively) we conclude that about 10^4 times more $p_{3/2}$ -state electrons are emitted when compared with emission of $s_{1/2}$ -state electrons. The reasons for it are as follows: i) $F_1(Z, E_e) \gg F_0(Z, E_e)$ for $E_e - m_e < Q$ (see Appendix B), ii) the maximal momentum of electron (~ 49.3 keV) is much larger than the maximal momentum of the neutrino (~ 2.5 keV). Henceforth, we shall neglect a small contribution to the differential decay rate given by an emission of the $s_{1/2}$ -state electrons.

For a normal hierarchy (NH) of neutrino masses with $m_3 > m_2 > m_1$ the Kurie function of the β -decay of

^{187}Re is given by

$$\begin{aligned} K(y) &= \sqrt{\frac{d\Gamma/dE_e}{p_e E_e F_0(Z, E_e)}} \\ &= \mathcal{B}_{Re} \sqrt{y + m_1} \left[|U_{e1}|^2 \sqrt{y(y + 2m_1)} \right. \\ &\quad + |U_{e2}|^2 \sqrt{(y + m_1 - m_2)(y + m_1 + m_2)} \times \\ &\quad \theta(y + m_1 - m_2) \\ &\quad \left. + |U_{e3}|^2 \sqrt{(y + m_1 - m_3)(y + m_1 + m_3)} \times \right. \\ &\quad \left. \theta(y + m_1 - m_3) \right]^{1/2}, \end{aligned} \quad (5)$$

with

$$\mathcal{B}_{Re} = \frac{G_F V_{ud} \sqrt{B}}{\sqrt{2\pi^3}} \sqrt{\frac{R^2 p_e^2}{3} \frac{F_1(Z, E_e)}{F_0(Z, E_e)}} \quad (6)$$

and $y = (E_0 - E_e - m_1) \geq 0$. The ratio $(p_e^2 F_1(Z, E_e)) / F_0(Z, E_e)$ depends only weakly on the electron momentum in the case of the β -decay of rhenium. With a good accuracy the factor \mathcal{B}_{Re} can be considered to be a constant.

The current upper limit on neutrino mass from tritium and rhenium β -decay experiments holds in the degenerate neutrino mass region ($m_1 \simeq m_2 \simeq m_3 \simeq m_0$ with $m_0 = \sum_{i=1}^3 m_i/3$) for which $m_\beta \simeq m_0$. So far the future rhenium β -decay experiments will not see any effect due to a smallness of neutrino masses, it is possible to approximate $m_k \ll Q - T$ ($k=1,2,3$) and obtain

$$K(y) \simeq \mathcal{B}_{Re} \left((y + m_\beta) \sqrt{y(y + 2m_\beta)} \right)^{1/2}, \quad (7)$$

where $y = (E_0 - E_e - m_\beta)$. In Fig. 2 we show the Kurie plot for β -decay of ^{187}Re versus y near the endpoint. We see that the Kurie plot is linear near the endpoint for $m_\beta = 0$. However, the linearity of the Kurie plot is lost if m_β is not equal to zero.

We note that there is also a possibility of bound-state decay of ^{187}Re , where an electron is placed into a bound state above occupied electron shells of the final atom. In Ref. [5] it was shown that the ratio of decay rates of bound-state and continuum β -decays of ^{187}Re is less than 1%.

III. CONCLUSIONS

For the first unique forbidden β -decay of ^{187}Re to ground state of ^{187}Os the theoretical spectral shape is presented. The decay rate of the process is a sum of particular decay rates associated with emission of $s_{1/2}$ and $p_{3/2}$ electrons, which depend in a different way on the neutrino mass. The p-wave emission is dominant over the s-wave. So, the Kurie function, defined by Eq. (6), is almost linear in the endpoint region. An observed deviation from the linearity indicates effects of the finite neutrino mass.

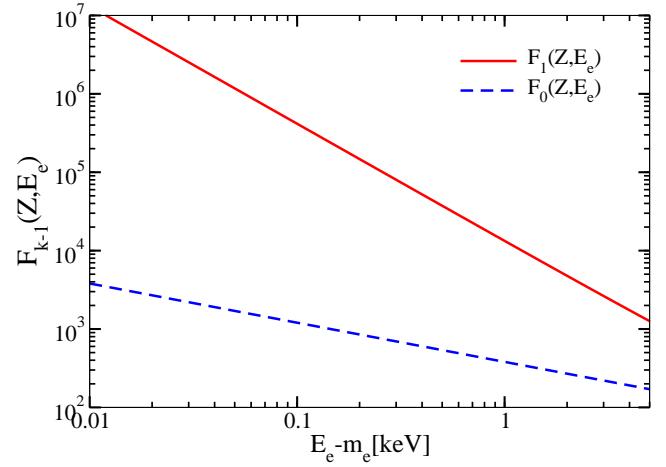


FIG. 3. (Color online) Relativistic Fermi functions $F_0(Z, E_e)$ and $F_1(Z, E_e)$ vs. electron energy E_e for the β -decay of ^{187}Re .

The analysis of the the Kurie function of the first forbidden β -decay of ^{187}Re show that close to the endpoint it coincides up to a factor to the Kurie function of super-allowed β -decay of tritium [6]. These findings are important for the planned MARE experiment, which will be sensitive to neutrino mass in the sub eV region.

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Appendix A: Relativistic electron wave functions

We adopt the relativistic electron wave function in a uniform charge distribution in a nucleus, which is expanded in terms of spherical waves:

$$\Psi(E_e, \mathbf{r}) = \Psi^{(s_{1/2})}(E_e, \mathbf{r}) + \Psi^{(p_{1/2})}(E_e, \mathbf{r}) + \Psi^{(p_{3/2})}(E_e, \mathbf{r}) + \dots \quad (\text{A1})$$

where of particular interest are $s_{1/2}$ and $p_{3/2}$ states [7]:

$$\begin{aligned} \Psi^{(s_{1/2})}(E_e, \mathbf{r}) &= \begin{pmatrix} \tilde{g}_{-1}(r)\chi_s \\ \tilde{f}_{+1}(r)(\sigma \cdot \hat{\mathbf{p}})\chi_s \end{pmatrix} \\ \Psi^{(p_{3/2})}(E_e, \mathbf{r}) &= i \begin{pmatrix} \tilde{g}_{-2}(r)[3(\hat{\mathbf{r}} \cdot \hat{\mathbf{p}}) - (\sigma \cdot \hat{\mathbf{r}})(\sigma \cdot \hat{\mathbf{p}})]\chi_s \\ \tilde{f}_{+2}(r)[(\hat{\mathbf{r}} \cdot \hat{\mathbf{p}})(\sigma \cdot \hat{\mathbf{p}}) - (\sigma \cdot \hat{\mathbf{r}})]\chi_s \end{pmatrix}. \end{aligned} \quad (\text{A2})$$

Here, \mathbf{r} is a position vector, $r = |\mathbf{r}|$, $\hat{\mathbf{r}} = \mathbf{r}/r$ and $\hat{\mathbf{p}} = \mathbf{p}_e/p_e$. By keeping the lowest power in expansion of r the radial wave functions take the form:

$$\begin{pmatrix} \tilde{g}_{-1}(r) \\ \tilde{f}_{+1}(r) \end{pmatrix} = \tilde{A}_{\mp 1}, \quad \begin{pmatrix} \tilde{g}_{-2}(r) \\ \tilde{f}_{+2}(r) \end{pmatrix} = \tilde{A}_{\mp 2}(pr/3). \quad (\text{A3})$$

In approximation up to $(\alpha Z)^2$ terms we have

$$\tilde{A}_{\pm k} \simeq \sqrt{(E_e \mp m_e)/(2E_e)} \sqrt{F_{k-1}(Z, E_e)}. \quad (\text{A4})$$

Appendix B: Relativistic Fermi functions

Relativistic Fermi function $F_{k-1}(Z, E_e)$ takes into account the distortion of electron wave function due to electromagnetic interaction of the emitted electron with the atomic nucleus. It takes the form [7]:

$$F_{k-1}(Z, E_e) = \left(\frac{\Gamma(2k+1)}{\Gamma(k)\Gamma(1+2\gamma_k)} \right)^2 (2p_e R)^{2(\gamma_k-k)} \times |\Gamma(\gamma_k + iz)|^2 e^{\pi z} \quad (\text{B1})$$

where $k = 1, 2, 3, \dots$ and

$$\begin{aligned} \gamma_k &= \sqrt{k^2 - (\alpha Z)^2} \\ z &= \alpha Z \frac{E_e}{p_e}. \end{aligned} \quad (\text{B2})$$

Following Ref. [8] it can be written as

$$F_{k-1}(Z, E_e) = C_{k-1} d_{k-1}(E_e) \left(\frac{m_e}{p_e} \right)^{2k-1} \left(\frac{E_e}{m_e} \right)^{2\gamma_k-1}. \quad (\text{B3})$$

The constant C_{k-1} is given by

$$C_{k-1} = 2\pi\alpha Z (2\alpha Z m_e R)^{2(\gamma_k-k)} \left[\frac{2k(2k-1)!!}{\Gamma(1+2\gamma_k)} \right]^2. \quad (\text{B4})$$

The function d_{k-1} takes the form

$$d_{k-1}(E_e) = \frac{1}{2\pi} z^{1-2\gamma_k} e^{\pi z} |\Gamma(\gamma_k + iz)|^2. \quad (\text{B5})$$

We note that

$$\lim_{p_e \rightarrow 0} d_{k-1}(E_e) = 1. \quad (\text{B6})$$

The Fermi functions $F_0(Z, E_e)$ and $F_1(Z, E_e)$ are related to the emission of $s_{1/2}$ and $p_{3/2}$ electrons, respectively. In Fig. 3, they are plotted as functions of the kinetic energy of electrons emitted in β -decay of ^{187}Re . We note that values of $F_1(Z, E_e)$ are significantly larger than those of $F_0(Z, E_e)$.

A quantity of interest is the ratio

$$\begin{aligned} \frac{p_e^2 F_1(Z, E_e)}{F_0(Z, E_e)} &= \frac{C_1}{C_0} m_e^2 \left(\frac{E_e}{m_e} \right)^{2(\gamma_2-\gamma_1)} \\ &\simeq \frac{C_1}{C_0} m_e^2 \left[1 + 2 \frac{(E_e - m_e)}{m_e} \right]. \end{aligned} \quad (\text{B7})$$

As the Q -value of rhenium β -decay is only 2.47 keV the above ratio can be to a good accuracy considered to be a constant with the value $0.11 m_e^2$.

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